Stats Review
chapters 7-8

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Examples are taken from Statistics 4 E by Michael Sullivan, III
And the corresponding Test Generator from Pearson
Note:

This review is composed of questions the textbook and the test generator. This review is meant to highlight basic concepts from the course. It does not cover all concepts presented by your instructor. Refer back to your notes, unit objectives, handouts, etc. to further prepare for your exam. A copy of this review can be found at [www.sctcc.edu/cas](http://www.sctcc.edu/cas). The final answers are displayed in red and the chapter/section number is the corner.
Uniform Distribution

The diameter of ball bearings produced in a manufacturing process can be explained using a uniform distribution over the interval 7.5 to 9.5 millimeters. What is the probability that a randomly selected ball bearing has a diameter greater than 8.6 millimeters.

1) Draw the distribution
2) Calculate the height
   \[ \text{height} = \frac{1}{\text{max} - \text{min}} = \frac{1}{9.5 - 7.5} = \frac{1}{2} \]
3) Shade the desired range
4) Find the area of the shaded range (length times height)
   \[ (9.5 - 8.6)(1/2) = 0.45 \]
   This is the probability.
Find z-value

For a standard normal curve, find the z-value, $z_{\alpha}$, $Z_{\alpha/2}$

- Area to the left is .1
  - Look for where the probability is .1, $z = -1.28$
- Area to the right is .25
  - Area is to the right so first $1-.25=.75$
  - The $z$-value where the probability is .75 is .67
- $Z_{.01}$
  - This is the same as asking to find the $z$-value where the area to the right is .01
  - $1-.01=.99$; The $z$-value where the probability is .99 is 2.33
- For 93% Confidence
  - This is asking for $Z_{\alpha/2}$
  - $\frac{1-.93}{2} = 0.035$
    - Can look up this probability where the $z$-value would be -1.81 and take the positive so 1.81
    - Can add 0.035 to .93 to get .965, look up this probability, $z = 1.81$
Normal Distribution Probability

A study found for Wendy’s the mean time spent in the drive-through was 138.5 seconds. Assuming the drive-through times are normally distributed with a standard deviation of 29 seconds.

a) $P(2 < x < 3)$?

b) Would it be unusual for a car to spend more than 3 minutes in the drive-through? Why?
Normal Distribution Probability

a) \( P(2 < x < 3) \)?

This is asking what is the proportion of cars spend between 2 and 3 minutes in a Wendy’s drive-through.

2 minutes = 120 seconds, 3 minutes = 180 seconds

1) Find the z-score for both 120 seconds and 180 seconds.

\[
\begin{align*}
    z &= \frac{x-\mu}{\sigma} = \frac{120-138.5}{29} = -0.64 \\
    z &= \frac{x-\mu}{\sigma} = \frac{180-138.5}{29} = 1.43
\end{align*}
\]

2) Find the probabilities of both these values

Probability associated with -0.64 is 0.2611

Probability associated with 1.43 is 0.9236

3) Since we want the proportion of cars between 2 and 3 minutes, we will subtract the probabilities from step 3.

0.9236 - 0.2611 = 0.6625

Conclusion: About 0.6625 of cars spend between 2 and 3 minutes in the drive-through.
Normal Distribution Probability

b) Would it be unusual for a car to spend more than 3 minutes in the drive-through? Why?

3 minutes = 180 seconds

1) Find the z-score
\[ z = \frac{x - \mu}{\sigma} = \frac{180 - 138.5}{29} = 1.43 \]

2) Find the probabilities of this value

   Probability associated with 1.43 is .9236

3) We want to know if it is unusual to spend more than 3 minutes in the drive through, we need to subtract the probability from 1. 1 - .9236 = .0764.

4) Since the probability of spending more than 3 minutes in the drive-through is greater than 5% (or .05), it is not unusual to spend more than 3 minutes in the drive-through.
Finding the Value Given the Probability

The mean incubation time of fertilized chicken eggs kept at 100.5°F in an incubator is 21 days. Suppose that the incubation times are approximately normally distributed with a standard deviation of 1 day.

a) Determine the 17th percentile for incubation time for fertilized eggs.

b) Determine the incubation times that make up the middle 93% of fertilized eggs.
Finding the Value Given the Probability

a) Determine the 17\(^{th}\) percentile for incubation time for fertilized eggs.

1) First find the z-score that corresponds to .17. Look it up in the table or use technology.
   
   \[ z = -0.95. \]

2) Find \( x \) using the formula \( x = z\sigma + \mu \)

   \[ x = (-0.95)(1) + 21 = 20.05 \text{ days} \]
Finding the Value Given the Probability

b) Determine the incubation times that make up the middle 93% of fertilized eggs.

1) Find the z-scores for the middle 93%. This is the same as finding $z_{\alpha/2}$

The bottom percent is $\frac{1-0.93}{2} = 0.035$. Looking this value up we find a z-score of -1.81. Using symmetry the other z-score is 1.81. We need both since we are looking for the middle

2) Find both values of $x$ using the formula $x = z\sigma + \mu$

$x = (-1.81)(1) + 21 = 19.19 \text{ days}$
$x = (+1.81)(1) + 21 = 22.81 \text{ days}$

19.19-22.81 days make up the middle 93% percent
Assessing Normality

Determine whether the normal probability plot indicates that the sample data could have come from a population that is normally distributed.

**Not Normal**

**Normal**

![Probability Plot of Time](image1)

![Probability Plot of FEV/VC](image2)
The Normal Approximation to the Binomial Probability Distribution

- Must satisfy \( np(1-p) \geq 10 \)
  - The mean: \( \mu_x = np \)
  - The Standard Deviation: \( \sigma_x = \sqrt{np(1-p)} \)

<table>
<thead>
<tr>
<th>Exact Probability Using Binomial</th>
<th>Approximate Probability Using Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(a) )</td>
<td>( P(a-.05 \leq X \leq a+.05) )</td>
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<tr>
<td>( P(X \leq a) )</td>
<td>( P(X \leq a+.05) )</td>
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<tr>
<td>( P(x &lt; a) )</td>
<td>( P(X \leq (a-1)+.05) )</td>
</tr>
<tr>
<td>( P(X \geq a) )</td>
<td>( P(X \geq a-.05) )</td>
</tr>
<tr>
<td>( P(X &gt; a) )</td>
<td>( P(X \geq (a+1)-.05) )</td>
</tr>
<tr>
<td>( P(a \leq X \leq b) )</td>
<td>( P(a-.05 \leq X \leq b+.05) )</td>
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</table>
The Normal Approximation to the Binomial Probability Distribution

A local rental car agency has 100 cars. The rental rate for the winter months is 60%. Find the probability that in a given winter month at least 70 cars will be rented.

1) Can we use normal approximation
   Yes; 100(.6)(1-.4)≥10

2) Find the mean and standard deviation
   \[ \mu_X = 100(.6) = 60; \sigma_X = \sqrt{100(.6)(1-.4)} = 4.899 \]

3) Want \( P(X\geq70) \). According to the Previous slide this is \( P(X\geq70-.05)=P(X\geq79.5) \)

4) Find the z-score
   \[ z = \frac{x-\mu}{\sigma} = \frac{69.5-60}{4.899} = 1.94 \]

5) Look up the probability: probability is .9738

6) Subtract from 1: 1-.9738=.0262

The probability that in a given winter month at least 70 cars will be rented is .0262.
Based on tests of the Chevy Cobalt, engineers have found that the miles per gallon are normally distributed, with a mean of 32 miles per gallon and a standard deviation of 3.5 mile per gallon.

a) What is the probability that a randomly selected cobalt gets more than 34 miles per gallon.

b) Twenty Cobalts are randomly selected and the miles per gallon is recorded. What is the probability that the mean miles exceeds 34 miles per hour? Would this result be unusual?
Distribution of Sample Mean

a) What is the probability that a randomly selected cobalt gets more than 34 miles per gallon.

Find the z-value

\[
z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{34 - 32}{3.5/\sqrt{1}} = .57
\]

The corresponding probability is .7157. Since we want the probability that the car gets more than 34 miles per gallon, we need to subtract from 1.

1-.7157=.2843

Thus the probability is .2843.
b) Twenty Cobalts are randomly selected and the miles per gallon is recorded. What is the probability that the mean miles exceeds 34 miles per hour? Would this result be unusual?

Find the z-score

\[ z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{34 - 32}{3.5 / \sqrt{20}} = 2.56 \]

Find the probability associated with a z-value of 2.56. The probability is .9949. We want the probability it exceeds 34, we need to subtract from 1. 1-.9949=.0051.

**The probability is .0051.**

Since the probability is less than 5%, the result would be unusual.
Describe a Sample Distribution

A simple random sample of size 16 is obtained from a population with $\mu=14$ and $\sigma=3$. What is the sampling distribution of $\bar{x}$?

1. $\mu_{\bar{x}} = \mu = 14$

2. $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{16}} = .75$

The sampling distribution of $\bar{x}$ is normal with $\mu_{\bar{x}} = 14$ and $\sigma_{\bar{x}} = .75$. 

8.1
Distribution of Sample Proportion

The National Association of Retailers estimates that 23% of all homes purchased were investment properties. If a sample of 800 homes sold was obtained, what is the probability that at most 200 homes are going to be used as an investment property.

1) Find $\hat{p}$. $\hat{p} = \frac{x}{n} = \frac{200}{800} = .25$

2) Identify $\mu_\hat{p} = \mu, \sigma_\hat{p} = \sigma$. $\sigma_\hat{p} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.23(1-.23)}{800}}$

3) Find the z-value.

$$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}} = \frac{.25 - .23}{\sqrt{.23(1-.23)/800}} = 1.34$$

4) Find the probability. The probability is $0.9099$. 